St. Xavier's College

(Intermediate Section), Ranchi

Worksheet: 1

(Intellectual Property of the College)

To be submitted after lockdown for

Internal assessment

Date: 14/05/2020

Date of submission: 21/05/2020

<table>
<tr><td colspan="2">St. Xavier's College
(Intermediate Section), Ranchi</td></tr>
<tr><td>Worksheet: 1</td><td>Mathematics</td></tr>
<tr><td colspan="2">Chapters :
Matrices, Determinants, Continuity , LPP & Probability</td></tr>
<tr><td>Forms of Questions</td><td>Very Short Answer, Short Answer & Long Answer Type</td></tr>
<tr><td>Estimated difficulty level</td><td>Average & Board level</td></tr>
</table>

Matrices

01. Express $\begin{bmatrix} 2 & 4 & -1 \\ 3 & 5 & 8 \\ 1 & -2 & 1 \end{bmatrix}$ **as the sum of a symmetric and a skew-symmetric matrix.**

02. Let $A=\begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$ **and** $B=\begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$

Verify that $(AB)^{-1}=B^{-1}A^{-1}$

03. Find A^{-1} **if** $A=\begin{bmatrix} -1 & 2 & 5 \\ 2 & -3 & 1 \\ -1 & 1 & 1 \end{bmatrix}$ **and hence, solve the system of linear equations**

$-x+2y+5z=2$

$2x-3y+z=15$

$-x+y+z=-3$

04. If $A=\begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$ $and\ B=\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$**, Compute (sin$\theta$) A + (cos$\theta$) B**

05. Given that $A=\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ **and** $B=\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$.

Find AB. Use this to solve the following system of questions: $\begin{aligned} x-y&=3 \\ 2x+3y+4z&=17 \\ y+2z&=7. \end{aligned}$

06. Find the inverse of the following matrix by using elementary transformations

$A=\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$.

07. If $A=\begin{bmatrix}1 & 2 & 2\\ 2 & 1 & 2\\ 2 & 2 & 1\end{bmatrix}$, **verify that $A^2 - 4A - 5I = 0$.**

08. **Let** $A=\begin{bmatrix}0 & -\tan(\alpha/2)\\ \tan(\alpha/2) & 0\end{bmatrix}$ **and I be the identity matrix of order 2.**

Show that $I + A = (I-A)\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$

09. **Prove that the product of matrices**

$$\begin{bmatrix}\cos^2\theta & \cos\theta\sin\theta\\ \cos\theta\sin\theta & \sin^2\theta\end{bmatrix} \text{ and } \begin{bmatrix}\cos^2\phi & \cos\phi\sin\phi\\ \cos\phi\sin\phi & \sin^2\phi\end{bmatrix}$$

is the null matrix , when θ *and* ϕ *differ by an odd multiple of* $\frac{\pi}{2}$

10. **If** $A=\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$ **and** $B=\begin{bmatrix}3 & 1 & -1\\ 1 & 3 & 1\\ -1 & 1 & 3\end{bmatrix}$, **find AB and use this result to solve the following system of equations:**

$$\begin{aligned}2x-y+z&=-1\\ -x+2y-z&=4\\ x-y+2z&=-3\end{aligned}$$

Determinants

11. **Prove that** $\begin{vmatrix}a^2+1 & ab & ac\\ ab & b^2+1 & bc\\ ac & bc & c^2+1\end{vmatrix} = \begin{vmatrix}a^2+1 & b^2 & c^2\\ a^2 & b^2+1 & c^2\\ a^2 & b^2 & c^2+1\end{vmatrix} = 1+a^2+b^2+c^2$

12. **Show that** $\begin{vmatrix}(a+b)^2 & a^2 & a^2\\ b^2 & (c+a)^2 & b^2\\ c^2 & c^2 & (a+b)^2\end{vmatrix} = 2abc(a+b+c)^3$

13. **Prove that :** $\begin{vmatrix}\alpha & \beta & \gamma\\ \alpha^2 & \beta^2 & \gamma^2\\ \beta+\alpha & \gamma+\alpha & \alpha+\beta\end{vmatrix} = (\beta-\gamma)(\gamma-\alpha)(\alpha-\beta)(\alpha+\beta+\gamma)$

14. **Show that** $\begin{vmatrix}1 & 1 & 1\\ a^2 & b^2 & c^2\\ a^3 & b^3 & c^3\end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca).$

15*. $\Delta = \begin{vmatrix} x & y & y \\ z & x & y \\ z & z & x \end{vmatrix} = \dfrac{z(z-y)^3 - y(x-z)^3}{z-y}$

16. Show that $\begin{vmatrix} (b+c)^2 & ba & ca \\ ab & (c+a)^2 & cd \\ ac & bc & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$

Continuity

17. **Let** $f(x) = \begin{cases} \dfrac{1-\cos ax}{x\sin x}, & x \neq 0 \\ \dfrac{1}{2}, & if\ x = 0 \end{cases}$ $, if\ f$ **is continuous at 0 find *a*.**

18. **Examine the continuity** $f(x) = \begin{cases} \dfrac{x-|x|}{x}, & x \neq 0 \\ 2, & if\ x = 0 \end{cases}$ $, at\ x = 0$

19. **Is** $f(x) = \begin{cases} (1+x)^{\frac{1}{x}}; when\, x \neq 0 \\ e \quad ; when\, x \neq 0 \end{cases}$ **Continuous at** $x = 0$?

20. **Find the values of *k* so that the function** $f(\alpha) = \begin{cases} kx+1, if\ x \leq 5 \\ 3x-5, if\ x > 5 \end{cases}$ **is continuous at** $x = 5.$

21. Determine a, b, c so that $f(x) = \begin{cases} \dfrac{\sin(a+1)x + \sin x}{x} & ; x < 0 \\ c & ; x = 0 \\ \dfrac{\sqrt{x+bx^2} - \sqrt{x}}{bx^{3/2}} & ; x > 0 \end{cases}$ **is continuous at *x* = *0***

22.Test the continuity of the function f(x) at x = 0 where $f(x) = \begin{cases} \dfrac{\sin x}{x} & ; x \neq 0 \\ 2 & ; x = 0 \end{cases}$

23.Test the continuity of the function f(X) at *x* = 2 if $f(x) = \begin{cases} \dfrac{x^2-4}{x-2} & ; x \neq 2 \\ 2 & ; x = 2 \end{cases}$

24.Test the continuity of the function f(x) at x = 0 where $f(x) = \dfrac{\sin x}{x}$ **when** $x \neq 0 = 2$ **when** $x = 0$

25.Solve graphically the following L.P.P.:

Minimize ***Z=5x+10y***

Subject to $x+y \geq 60$, $x+2y \geq 120$, $x-2y \geq 0$ **and** ***x-2y ≥ 0***

26.Solve graphically the following L. P. P.:

Minimize Z = -3x + 4y

Subject to x + 2y $\leq$ 8

3x + 2y $\leq$ 12

and x, y $\geq$ 0.

27. Solve the following LLP graphically :

Maximize $z = 3x + 2y$

Subject to $x + 2y \leq 10$, $3x + y \leq 15$

and $x, y \geq 0$

28. Solve the following LLP graphically :

Maximize $Z = 5x + 3y$ **subject to the constraints**

$3x + 5y \leq 15$

$5x + 2y \leq 10$

and $x, y \geq 0$

29. A speaks truth in 75% cases and B in 80% of the cases. In what percent are they likely to contradict each other in narrating the same statement.

30. If $P(A)=\frac{2}{5}, P(B)=\frac{1}{3}$ **and** $P(A\cup B)=\frac{1}{2},$ **then**

find $P\left(\frac{A}{B}\right)$ **and** $P\left(\frac{B}{A}\right).$

31. If $P(A)=0.6, P(B)=0.3$ **and** $P(A\cap B)=0.1,$ **find** $P(A'\cap B').$

32. A bag A contains 3 white and 2 red balls and another bag B contains 5 white and 4 real balls. One ball is drawn at random from one of them bags and it is found

to be red. What is the probability that it was drawn from the bag B?

33. A bag contains 3 white and 2 black balls. Find the probability of drawing a white ball at random.

34. Let A and B be two events such that $2P(A)=P(B)=\frac{5}{13}$ **and** $P(A/B)=\frac{2}{5}.$ **Find** $P(A\cup B).$

35. A manufacturer has three machine operators, *A*,*B* and *C*. The first operator *A* produces 1% defective items, whereas the other two operations *B* and *C* produces 5% and 7% defective items respectively. *A* is on the job for 50% of the time, *B* on the job for 30% of the time and *C*

on the job for 20% of the time. Let E_1:be event that an item is produced by *A*, E_2 :be event that an item is

produced by *B* and E_3: be event that an item isproduced by *C*. If *E* be the event that a defective item is produced, them find the following:

(i) $P(E_1)$ **(ii)** $P(E_2)$ **(iii)** $P(E_3)$

(iv) $P(E/E_1)$ **(v)** $P(E/E_1)$ **(vi)** $P(E_1/E)$